Synchronized chaotic intermittent and spiking behavior in coupled map chains

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We study phase synchronization effects in a chain of nonidentical chaotic oscillators with a type-I intermittent behavior. Two types of parameter distribution, linear and random, are considered. The typical phenomena are the onset and existence of global (all-to-all) and cluster (partial) synchronization with increase of coupling. Increase of coupling strength can also lead to desynchronization phenomena, i.e., global or cluster synchronization is changed into a regime where synchronization is intermittent with incoherent states. Then a regime of a fully incoherent nonsynchronous state (spatiotemporal intermittency) appears. Synchronization-desynchronization transitions with increase of coupling are also demonstrated for a system resembling an intermittent one: a chain of coupled maps replicating the spiking behavior of neurobiological networks.

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I. INTRODUCTION

The study of cooperative behavior in ensembles of chaotic oscillators is a topical problem of nonlinear dynamics. Chaotic synchronization in such spatially extended systems has been considered for populations of locally and globally coupled maps [1–8] as well as for ensembles of locally and globally coupled continuous-time chaotic oscillators [9–14]. The theoretical knowledge obtained has been often applied to describe dynamical processes in various biological and physical systems. In spatially extended systems the effect opposite to synchronized oscillations is spatiotemporal disorder, in particular spatiotemporal intermittency (STI). It is one of the most fascinating phenomena appearing in a wide range of extended systems in several experimental situations, such as chemical reactions [15], Rayleigh-Bénard convection [16], planar Couette flow [17], fluid flows between rotating electrical cylinders [18], Taylor-Couette flows [19], etc., as well as in theoretical models, such as coupled map lattices [20] or partial differential equations [21]. Among basic types of synchronization (complete and generalized) *chaotic phase synchronization* (CPS) is a subject of active investigations (see [22]). CPS in ensembles of locally coupled chaotic elements was first studied in chains of weakly diffusively coupled chaotic Rössler oscillators [11]. Time-discrete systems were also under study.

Synchronization phenomena in ensembles of locally coupled circle maps were considered in [7]. Many phenomena observed in populations of periodic oscillators were found there too, noting especially the formation of several clusters of mutually synchronized elements and global synchronization. The study of CPS requires the existence of equations for the evolution of phase variables (as for coupled Rössler oscillators or circle maps) or at least the existence of appropriate definition of phases [23]. However, there are so far no unambiguous methods to obtain such equations and definitions. But in some cases specific properties of the chaotic attractors allow one to define the phases of chaotic oscillations in a rather simple way. Besides oscillators, where chaos appears through a period doubling cascade, it is possible to introduce a suitable phase for typical systems with intermittentlike behavior, especially for systems with type-I intermittent chaotic oscillations, or spiking neurons [24]. In this paper we investigate the collective dynamics in chains of such maps. Our study is motivated by the high importance of understanding mechanisms behind the transition from low-dimensional chaos (which may correspond to synchronized chaotic systems) to developed (spatiotemporal) turbulence which often looks like intermittent chaotic behavior.

The paper is organized as follows. In Sec. II we briefly describe the behavior of the quadratic map generating chaotic type-I intermittent behavior, introduce definitions of the phase and the frequency of oscillations, and give criteria for synchronization in chains of coupled maps. Synchronization phenomena as well as synchronization-desynchronization transitions with linear and random distributions of the control parameter are discussed in Secs. III and IV. In Sec. V we present results of our numerical study of chaotic phase synchronization in a chain of coupled spiking maps. The results are summarized in Sec. VI.

II. MODEL OF COUPLED INTERMITTENT MAPS. PHASE AND FREQUENCY. SYNCHRONIZATION CRITERIA

In the focus of this study is the synchronization problem in chains of coupled nonidentical maps with intrinsic type-I intermittent chaotic behavior. In order to measure the degree of synchronized motion, we will first introduce the frequency and phase of intermittent oscillations. Chaotic intermittent motion has a distinct *characteristic time scale* (CTS). For type-I intermittency a very large laminar stage (with duration τ) is followed by a very short turbulent stage (with duration T) and then the next laminar stage begins. Sometimes (for example, in the model map studied below) the turbulent stage has only one jump from a practically fixed variable

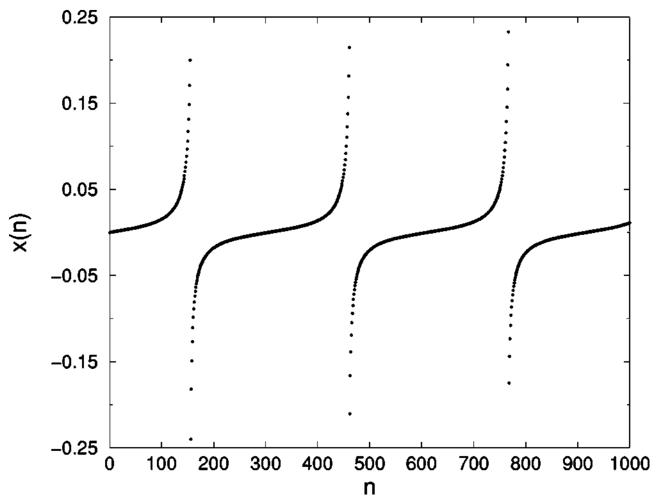

FIG. 1. Intermittent chaotic oscillations in a single quadratic map (6) and (7). Parameters are $\varepsilon=0.0001$, $g=2$.

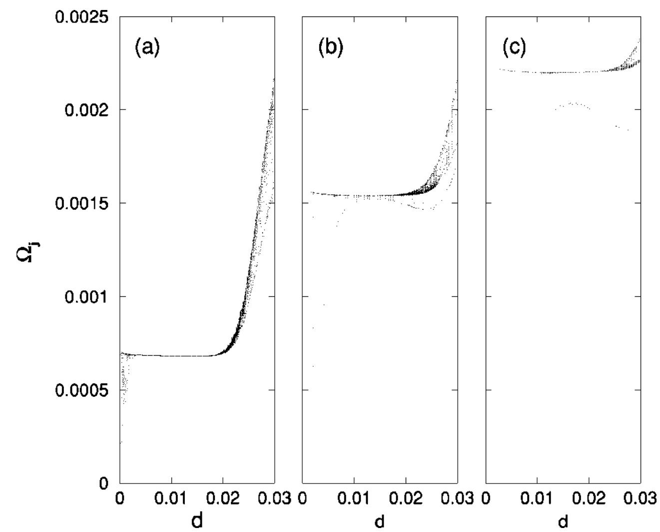

FIG. 2. The evolution of Ω_j [Eq. (4)] in dependence on coupling for $\varepsilon=0.000\,001$ and for three different values of $\Delta\varepsilon$ in a chain of 50 coupled maps. $\Delta\varepsilon=$ (a) 0.000 001; (b) 0.000 005; (c) 0.000 01.

value and back. This event is reminiscent of firing—a special behavior, which is typical for neuronal systems. Regarding this specific character of behavior we will distinguish between the laminar and the firing stages. The average length of the laminar stage for a single element is defined as [25]

$$\langle \tau_0 \rangle \propto \frac{1}{\sqrt{\varepsilon - \varepsilon^{\rm cr}}}, \tag{1}$$

where ε is a bifurcation parameter and $\varepsilon^{\rm cr}$ is the critical value when chaos sets in. For the coupled maps studied below the CTS $\langle T_c \rangle = \langle \tau + T \rangle$ can be calculated numerically as

$$\langle T_c \rangle = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} (k_{l+1} - k_l), \tag{2}$$

where k_l is the moment when the lth laminar stage sets in or in other words when the lth firing occurs. We note that in the studied maps because $\tau/T \gg 1$ the time of full cycle $T_c \sim \tau$, i.e., the time between the beginning of two sequential laminar stages, almost equals τ. Therefore, coincidence of the averaged τ leads to coincidence of the averaged T_c. One can also introduce a *phase of intermittent oscillations*, attributing to each interval between beginnings of the laminar stage (or in other words between two firings) a 2π phase increase:

$$\varphi^k = 2\pi \frac{k - k_l}{k_{l+1} - k_l} + 2\pi l, \quad k_l \leq k < k_{l+1}, \tag{3}$$

where k is discrete time.

The presence of a CTS and a suitable phase allows us to formulate the problem of chaotic phase synchronization in ensembles of coupled units with intermittent behavior. So, if the CTSs $\langle \tau_j \rangle$ or the corresponding frequencies

$$\Omega_j = 2\pi / \langle \tau_j \rangle \tag{4}$$

of all units become equal, this manifests their global 1:1 *frequency entrainment*. If the conditions

$$|\varphi_l^k - \varphi_m^k| < \text{const} \tag{5}$$

for all k are satisfied, one can speak about a 1:1 *phase locking* between the lth and the mth units.

Let us demonstrate mutual phase synchronization of chaotic intermittent oscillations for a chain of diffusively locally coupled nonidentical quadratic one-dimensional maps:

$$x_j^{k+1} = f_j(x_j^k) + d(x_{j-1}^k - 2x_j^k + x_{j+1}^k),$$
$$j = 1, \ldots, N, \tag{6}$$

where N is the number of elements in the chain, and $f_j(x)$ consists of the standard quadratic part that produces a laminar motion and a somewhat arbitrarily chosen return part that acts as a firing stage:

$$f_j(x) = \begin{cases} \varepsilon_j + x + x^2 & \text{if } x \leq 0.2, \\ g(x - 0.2) - \varepsilon_j - 0.24 & \text{if } x > 0.2. \end{cases} \tag{7}$$

Here g regulates the coherence properties of the chaotic attractor. In the case $g < 5$ the laminar stage duration is distributed in a rather narrow band, i.e., the chaotic behavior is highly coherent, but for $g > 5$ this distribution is rather broad. We will focus on the case of a coherent chaotic attractor and set $g=2$. We recall that the uncoupled map [$d=0$ in Eq. (6)] demonstrates a type-I intermittent behavior for $\varepsilon_j > 0$, i.e., $\varepsilon_j^{\rm cr} = 0$. Figure 1 shows a typical motion of the considered map.

The parameter ε_j defines the CTS in the individual jth oscillator. In our study we treat two cases: (i) a linear distribution of the parameter ε_j, $\varepsilon_j = \varepsilon_1 + \Delta\varepsilon(j-1)$, where $\Delta\varepsilon$ is the parameter mismatch between neighboring elements, and (ii) a random uniform distribution of natural frequencies in the range $[\varepsilon_1, \varepsilon_1 + \Delta\varepsilon(N-1)]$. We assume free-end boundary conditions:

$$x_0^k = x_1^k, \quad x_{N+1}^k = x_N^k \tag{8}$$

for all k.

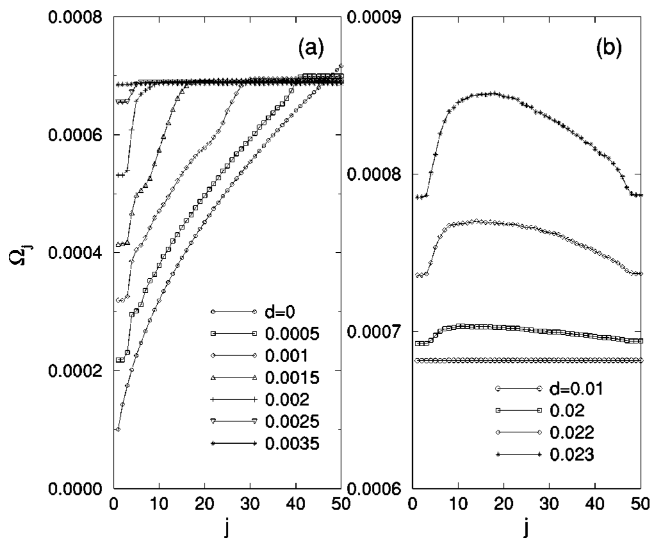

FIG. 3. The evolution of the observed frequencies Ω_j for different couplings for (a) the transition desynchronization-synchronization and (b) the transition synchronization-desynchronization. $\varepsilon = 0.000\,001$, $\Delta\varepsilon = 0.000\,000\,1$, and $N = 50$.

III. LINEARLY DISTRIBUTED CONTROL PARAMETER. SOFT TRANSITION TO GLOBAL SYNCHRONIZATION

First, a chain with a linear distribution of the parameters ε_j is explored. The evolution of the observed frequencies Ω_j in dependence on the coupling is presented in Fig. 2. In all diagrams with an increase of coupling from zero the tendency to a more coherent behavior is clearly seen. Then in dependence on the mismatch $\Delta\varepsilon$, global synchronization is observed [Fig. 2(a)] or is not [Figs. 2(b) and 2(c)]. But in all cases the increase of coupling leads to a fully incoherent behavior. The detailed analysis of the frequency distribution Ω_j vs coupling (see Fig. 3) shows that the transition to global synchronization is smooth, i.e., a gradual adjustment of frequencies is observed. The reason for such a "soft" route to global synchronization is the existence of two quite different time scales: the slow laminar stage and the fast firing stage. It is well known (see, for instance, [26]) that the appearance and interaction of many time scales (at least two) can lead in oscillatory systems to a chaotic behavior. Another consequence of the slow-fast motion is a large value of the frequency of global synchronization. It is close to the maximal individual frequency [27] [see Fig. 3(a)]. The reason for this effect is the following. The strong change (firing) of dynamical variable in the elements close to the end of the chain is faster than in other elements. For a sufficiently large coupling this provokes analogous strong change of the dynamical variable in the neighboring element which also provokes his neighbor, and so on. This process leads to a sequential firing in all elements in the chain. The transition to desynchronization appears also through a "soft" change of the observed frequencies. Corresponding results are presented in Fig. 3(b). A detailed analysis of synchronization-desynchronization transitions is presented for the case of a randomly distributed parameter ε_j in the next section.

IV. RANDOMLY DISTRIBUTED CONTROL PARAMETER. TRANSITION TO SPATIOTEMPORAL INTERMITTENCY

For randomly distributed ε_j, the evolution of the observed frequency distribution is shown in Fig. 4. Three types of transitions to global synchronization are observed here. (i) Two adjacent elements (clusters) with close frequencies can be easily synchronized and a new cluster appears. (ii) Nonlocal synchronization can occur, i.e., an element (a cluster of elements) becomes synchronized not to a nearest-neighbor element (cluster), but to some other element (cluster) having a close rotation number. Here, the observed frequencies of the elements (clusters) in between are considerably different. (iii) One element (group of elements) at the edge of one cluster can go to another neighboring cluster. Similarly to the case of linearly distributed parameters ε_j, in the case of random distribution of ε_j the regime of global synchronization can disappear with increase of the coupling. At some critical value d^* this regime becomes unstable. In the chain triangular embeddings are formed. The onset of such embeddings in some places in the chain leads to the propagation of firing

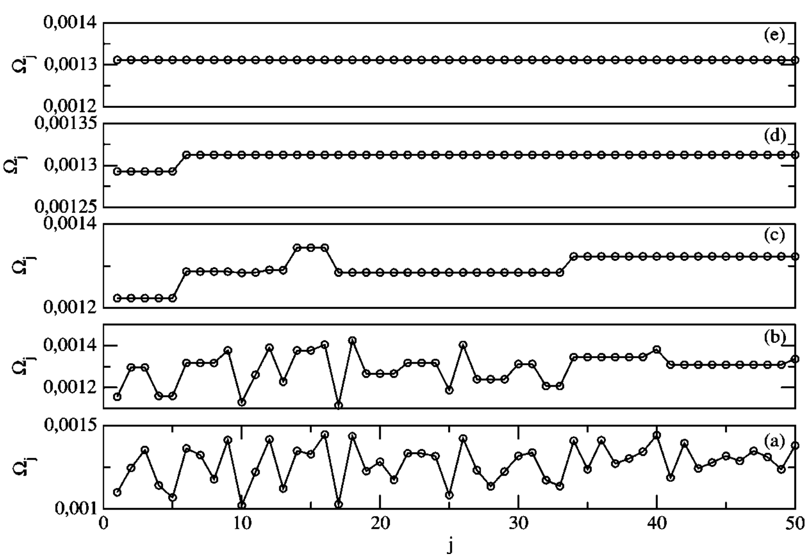

FIG. 4. The evolution of observed frequencies Ω_j [Eq. (4)] for different couplings d= (a) 0, (b) 0.0005, (c) 0.001, (d) 0.0015, and (e) 0.0025. $\varepsilon = 0.000\,001$, $\Delta\epsilon = 0.000\,000\,1$, and $N = 50$.

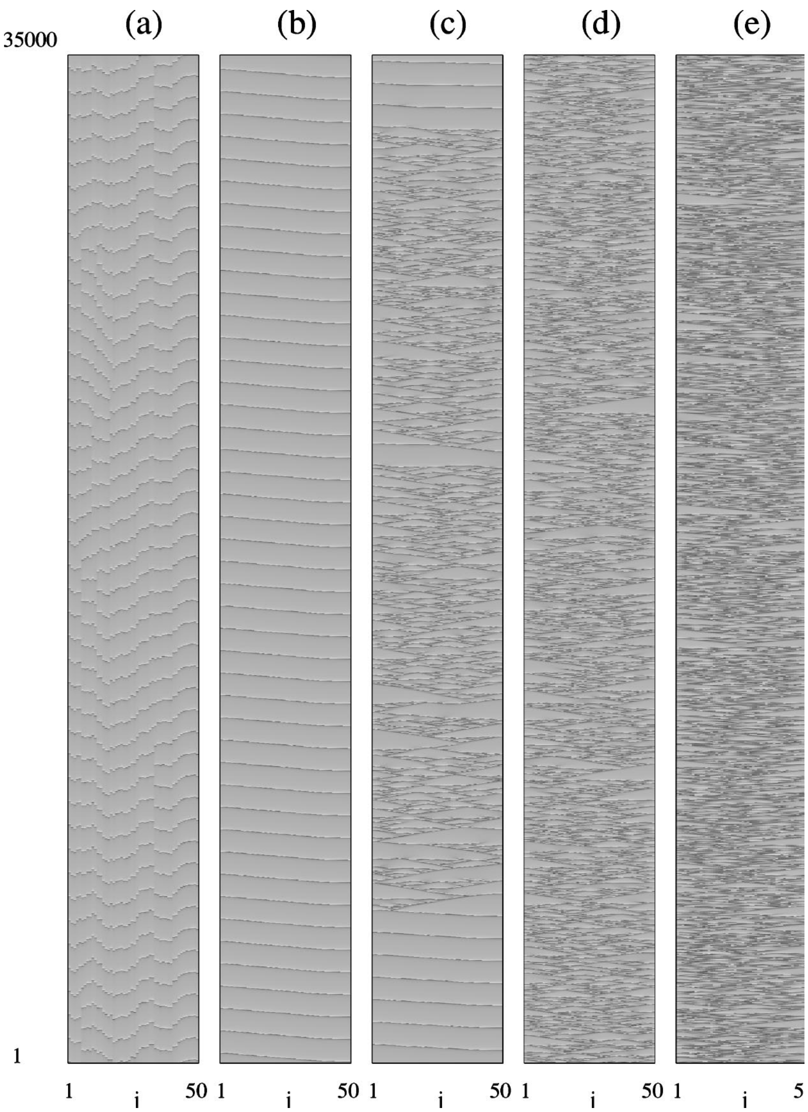

FIG. 5. Space-time plots of x_j for ε_j randomly distributed in the interval [0.000 005; 0.000 015]. (a) shows nonsynchronous regime at small coupling. Only some intervals of synchronous oscillations are seen. (b) corresponds to the regime of global synchronization. In (c) the intermittent regime of synchronous (in time intervals $t \in [0:5000]$ and $t \in [32\,000:35\,000]$) and nonsynchronous behaviors is shown. (d) and (e) present regimes of spatiotemporal intermittency. Parameters: $N=50$, $d=0.001$ (a), 0.04 (b), 0.0056 (c), 0.07 (d), and 0.15 (e).

processes in one or, more typically, in both directions. Propagating firing fronts are usually unstable and new triangular embeddings appear and this process repeats. Therefore the domains with a large synchronized intermittency are changed to domains of complex spatiotemporal behavior, which in the present context we call the spatially turbulent regime. This spatially turbulent regime appears suddenly and extends to the whole chain; then it suddenly disappears and in the whole chain the regime of synchronized intermittency is again realized. With an increase of coupling the duration of the spatially turbulent regime grows and correspondingly the duration of the synchronized regime becomes shorter. After some critical value d^{**}, the synchronized regime is no longer observed and the regime of fully developed spatiotemporal intermittency sets in. The rich spatiotemporal dynamics in the synchronous and nonsynchronous regimes is illustrated in Fig. 5. The left panel corresponds to nonsynchronous behavior (small values of coupling). There are several clusters of mutually synchronized elements. Only panel (b) corresponds to a synchronous regime. Panel (c) corresponds to the intermittency of synchronized and turbulent regimes. Panels (d) and (e) show highly developed STI. The tendency to the complication of collective oscillations with increase of coupling is clearly seen. In all plots the darker regions mark higher values of the presented variables.

It is interesting to analyze these observed processes by using our phase definition (3). Hence, we can state that in the regimes of perfect [Fig. 5(b)] and intermittent [Fig. 5(c)] chaotic phase synchronization, the phase distribution φ_j is a sequence of intervals with constant phase, separated by $\pm 2\pi$ kinks. The position of the kinks at constant time corresponds to a phase slip. In the synchronous regimes the phase slips appear with the frequency of synchronization. In the nonsynchronous regimes phase slips appear suddenly and rather fast.

In the presented model STI appears due to the relatively strong interaction of many units. The specific property in our observation consists in the existence of a transient regime from fully coherent (synchronous) to fully noncoherent (turbulent) behavior. In order to demonstrate this transition, we plot in Fig. 6 the ratio D of number of laminar stages corresponding to the synchronization regime and the full number of laminar stages. It is clearly seen that (i) for $d \gtrsim d^*$ the

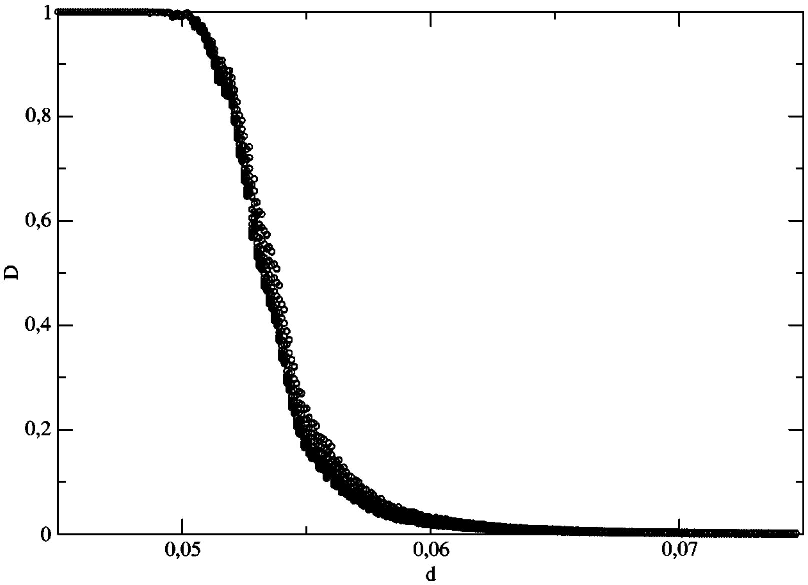

FIG. 6. The dependence of the ratio D on the coupling for a 50-element chain with ε_j randomly distributed in the interval [0.000 005; 0.000 015].

turbulent stages appear very rarely, and (ii) for $d \lesssim d^{**}$ there are very short intervals of laminar stages.

In our numerical study we also examined chains of different sizes and different boundary conditions, in particular periodic boundary conditions. Qualitatively all effects described above are the same.

V. COLLECTIVE OSCILLATIONS IN THE CHAIN OF SPIKING MAPS

There is a type of behavior often observed in neurobiological systems that resembles intermittency and is usually called "spiking." The rich collective dynamics of coupled intermittent systems urges analogous studies of neural ensembles. In simulations we next study a chain of locally coupled nonidentical model maps (replicating neural spiking activity) proposed in [28]:

$$x_j^{k+1} = f(x_j^k, x_j^{k-1}, y_j^k) + \frac{1}{2} d(x_{j+1}^k - 2x_j^k + x_{j-1}^k),$$

$$y_j^{k+1} = y_j^k - \mu(x_j^k + 1) + \mu\sigma_j + \mu\frac{1}{2} d(x_{j+1}^k - 2x_j^k + x_{j-1}^k),$$

$$j = 1, \ldots, N, \tag{9}$$

where x_j and y_j are the fast and slow variables respectively. $\mu=10^{-3}$, σ_j, and $\alpha=3.5$ are the parameters of the individual map, and d is the coupling. The function $f(\cdot,\cdot,\cdot)$ has the form

$$f(x^k, x^{k-1}, y^k) = \begin{cases} \alpha/(1 - x^k) + y^k & \text{if } x^k \leqslant 0, \\ \alpha + y^k & \text{if } 0 < x^k < \alpha + y^k \quad \text{and } x^{k-1} \leqslant 0, \\ -1 & \text{if } \quad x^k \geqslant \alpha + y^k \quad \text{or } x^{k-1} > 0. \end{cases} \tag{10}$$

In dependence on the parameters the individual dynamics of the map [in Eq. (9) $d=0$] ranges from a regular spiking to a chaotic spiking or bursting behavior and can, therefore, be used for the effective modeling of neuronlike elements. Several basic spatiotemporal regimes (including pulse and spiral wave propagation) for networks of identical maps (9) and (10) were presented in [8]. Here, we show synchronization phenomena in a chain of locally coupled *nonidentical* maps. As for maps with a type-I intermittent behavior the phase and frequency of oscillations can be defined by Eqs. (3) and (4), implying a 2π increase between subsequent spikes. Computer simulations show that as the coupling increases, three different kinds of spatiotemporal dynamics are observed. Similar to the case of a chain of intermittent maps, at small coupling neurons are spiking asynchronously [Fig. 7(a)], at medium coupling they synchronize [Fig. 7(b)], but at large coupling synchronization gets destroyed and spatiotemporal chaos sets in [Figs. 7(c) and 7(d)]. However, the nature of spatiotemporal chaos is different: initially rare spikes appear and act as phase slips or defects; further they evolve into chaotic bursts synchronized in phase [Figs. 7(d) and 7(e)]. Note that spikes forming these bursts are correlated in space, as they appear as triangular embedding with a fractal-like spatiotemporal structure. The transition observed shows how spiking maps can produce bursting behavior if they form a spatially extended system. Why does collective chaos differ for intermittent and spiking maps? This is due to the interplay between fast and slow dynamics that produces spiking behavior. The slow variable regulates the threshold value and when the threshold gets too high, it forces spike events to stop propagating along the chain and the burst ends. Until the fastest neuron is recovered, no spiking is observed in the chain and this quiescent state separates the bursts clearly. Quite on the contrary, there is no slow variable in the intermittent map that would regulate turbulent outbursts and they multiply freely in the regime of spatiotemporal chaos. A more detailed consideration of this phenomenon will be reported elsewhere.

VI. CONCLUSIONS

In conclusion, we have found the existence of global and cluster phase synchronization effects in a chain of noniden-

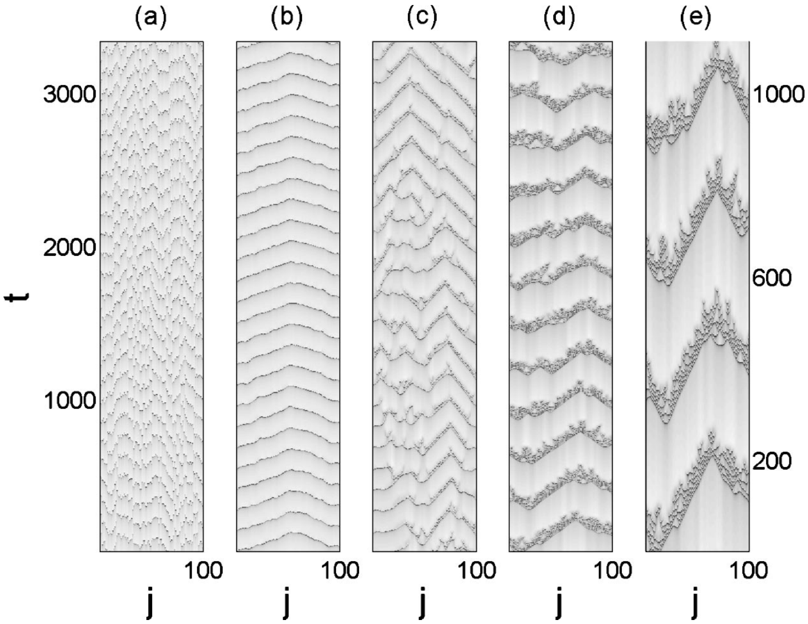

FIG. 7. Space-time plots of x_j for synchronous (b) and nonsynchronous regimes (a),(c),(d) for σ_j randomly distributed in the interval [0.15; 0.16]. $N=100$, $d=0.005$ (a), 0.05 (b), 0.09 (c), and 0.2 (d) and (e). (e) is an enlargement of a part of (d).

tical chaotic oscillators with type-I intermittent behavior. A very important feature is that an increase of the coupling strength can also lead to desynchronization phenomena, i.e., global or cluster synchronization is changed to a regime where synchronization is intermittent with the incoherent state. Then a regime of a fully incoherent nonsynchronous state, spatiotemporal intermittency, appears. Analogous synchronization phenomena, especially synchronization-desynchronization transitions with increase of coupling, have been observed in a chain of locally coupled nonidentical maps demonstrating spiking activity. It is important to note that the appearing chaotic traveling spikes (forming triangular embedding), which correspond to fully developed spatiotemporal intermittency, show nothing but space-time fractal bursting. Our results show that the transition to spatiotemporal intermittency is quite typical for intermittent systems discrete in time and space (see also [20]), which are often used for modeling of dynamical processes in oscillatory media. Obtained findings elucidate complex and intriguing collective dynamics of intermittent and spiking spatially extended systems, and may be potentially used in applied problems like developed (spatio-temporal) turbulence and complex behavior in neurobiological networks. We also expect experimental studies of these results in various fields, where type-I intermittency has been reported so far (see [29–35]).

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